XXVIII. *A new and general Method of finding ſimple and quickly-converging Series; by which the Proportion of the Diameter of a Circle to its Circumference may eaſily be computed to a great Number of Places of Figures. By* Charles Hutton, *Eſq. F. R. S.*

TO THE REV. DR. HORSLEY, SEC. R. S.

Royal Mil. Acad.
Jan. 25, 1776.

SIR,

R. May 1, 1776.

IN a late examination of the methods of Mr. MACHIN and others, for computing the proportion of the diameter of a circle to its circumference, I diſcovered the method which is explained in the paper accompanying this letter. This method you, SIR, will perceive is very general, and diſcovers many ſeries which are very fit for the abovementioned purpoſe. If you think it has ſufficient merit to entitle it to the honour of being offered to the Royal Society, I have taken the liberty to incloſe it to you, requeſting the favour of you to preſent it accordingly, from, &c.

P. S. In the courſe of the laſt year, I attended ſome very intereſting experiments on the effects of the force

force of fired gun-powder; as they ſeem to lead to many important concluſions, I believe I ſhall, in a ſhort time, have the honour of ſubmitting to your inſpection an account of ſome of them, for the like purpoſe as the following paper.

THE excellency of this method is primarily owing to the ſimplicity of the ſeries by which an arc is found from its tangent. For if t denote the tangent of an arc a, the radius being 1, then it is well known, that the arc a will be equal to the infinite ſeries,

$$\frac{t}{1} - \frac{t^3}{3} + \frac{t^5}{5} - \frac{t^7}{7} + \frac{t^9}{9} - \frac{t^{11}}{11} + \&c.$$

where the form is as ſimple as can be deſired. And it is evident, that nothing farther is required than to contrive matters ſo, as that the value of the quantity t in this ſeries may be both a ſmall and a very ſimple number. Small, that the ſeries may be made to converge ſufficiently faſt; and ſimple, that the ſeveral powers of t may be raiſed by eaſy multiplications, or eaſy diviſions.

Since the firſt diſcovery of the above ſeries, many have uſed it, and that after different methods, for determining the length of the circumference to a great number of figures. Among theſe were Dr. HALLEY, Mr. ABRAHAM SHARP, Mr. MACHIN, and others, of our own country; and M. DE LAGNY, M. EULER, &c. abroad. Dr. HALLEY uſed the arc of 30°, or $\frac{1}{12}$th of the circumference, the tangent of which being $=\sqrt{\frac{1}{3}}$, by ſubſtituting $\sqrt{\frac{1}{3}}$ in the above ſeries, and multiplying by 6, the ſemi circumference is $=6\sqrt{\frac{1}{3}}\times : 1-\frac{1}{3\cdot 3}+\frac{1}{5\cdot 3^2}-\frac{1}{7\cdot 3^3}+\frac{1}{9\cdot 3^4}$, &c. which ſeries is, to be ſure, very ſimple; but its rate of converging is not very great, on which account a great many terms muſt be uſed to compute the circumference to many places of figures. By this very ſeries, however, the induſtrious Mr. SHARP computed the circumference to 72 places of figures; Mr. MACHIN extended it to 100; and M. DE LAGNEY, ſtill by the ſame ſeries, continued it to 128 places of figures. But although this ſeries, from the 12th part of the circumference, does not converge very quickly, it is, perhaps, the beſt aliquot part of the circumference which can be uſed for this purpoſe; for when ſmaller arcs, which are exact aliquot parts, are uſed, their tangents, although ſmaller, are ſo much more complex, as to render them, on the whole, more operoſe in the application; this will eaſily appear, by inſpecting ſome inſtances, that have been given by Mr. GARDINER, in his editions of SHERWIN's Tables. One of theſe methods is from the arc of 18°, the tangent of which is $\sqrt{1-2\sqrt{\frac{1}{5}}}$; another is from the arc of $22°\frac{1}{2}$, the tangent of which is $\sqrt{2}-1$; and a third is from the arc of 15°, the tangent of which is $2-\sqrt{3}$. All of which are evidently too complex to afford an eaſy application to the general ſeries.

In order to a ſtill farther improvement of the method by the above general ſeries, Mr. MACHIN, by a very ſingular and excellent contrivance, has greatly reduced the labour

labour naturally attending it. His method is explained in Mr. MASERES's Appendix to his Diſſertation on the Uſe of the Negative Sign in Algebra; and I have given an analyſis of it, or a conjecture concerning the manner in which it is probable Mr. MACHIN diſcovered it, in my Treatiſe on Menſuration; which, I believe, are the only two books in which that method has been explained, as I never had ſeen it explained by any, till I met with Mr. MASERES's book abovementioned on the Uſe of the Negative Sign. For though the ſeries' diſcovered by that method were publiſhed by Mr. JONES, in his *Synopſis Palmariorum Matheſeos*, which was printed in the year 1706, he has given them merely by themſelves, without the leaſt hint of the manner in which they were obtained. The reſult ſhews, that the proportion of the diameter to the circumference is equal to that of 1 to quadruple the ſum of the two ſeries',

$$\begin{cases} \frac{4}{5} \times : 1 - \frac{1}{3 \cdot 5^2} + \frac{1}{5 \cdot 5^4} - \frac{1}{7 \cdot 5^6} + \frac{1}{9 \cdot 5^8}, \text{ \&c.} \\ \frac{1}{239} \times : 1 - \frac{1}{3 \cdot 239^2} + \frac{1}{5 \cdot 239^4} - \frac{1}{7 \cdot 239^6} + \frac{1}{9 \cdot 239^8}, \text{ \&c.} \end{cases}$$

The ſlower of which converges almoſt thrice as faſt as Dr. HALLEY's raiſed from the tangent of 30°. The latter of theſe two ſeries converges ſtill a great deal quicker; but then the large incompoſite number 239, by the reciprocals of the powers of which the ſeries converges, occaſions ſuch long, tedious diviſions, as to counter-balance its quickneſs of convergency; ſo that the former ſeries is ſummed, with rather more eaſe than the latter, to the ſame number

ber of places of figures. Mr. JONES, in his *Synopsis*, mentions other series' besides this, which he had received from Mr. MACHIN for the same purpose, and drawn from the same principle. But we may conclude this to be the best of them all, as he did not publish any other besides it.

M. EULER too, in his *Introductio in Analysin Infinitorum*, by a contrivance something like Mr. MACHIN's, discovers, that $\frac{1}{2}$ and $\frac{1}{3}$ are the tangents of two arcs, the sum of which is just 45°; and that, therefore, the diameter is to the circumference as 1 to quadruple the sum of the two series',

$$\begin{cases} \frac{1}{2} \times : 1 - \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 4^2} - \frac{1}{7 \cdot 4^3} + \frac{1}{9 \cdot 4^4}, \text{ \&c.} \\ \frac{1}{3} \times : 1 - \frac{1}{3 \cdot 9} + \frac{1}{5 \cdot 9^2} - \frac{1}{7 \cdot 9} + \frac{1}{9 \cdot 9^4}, \text{ \&c.} \end{cases}$$

Both which series' converge much faster than Dr. HALLEY's, and are yet at the same time made to converge by the powers of numbers producing only short divisions; that is, divisions performed in one line, or without writing down any thing besides the quotients.

I come now to explain my own method, which, indeed, bears some little resemblance to the methods of MACHIN and EULER; but then it is more general, and discovers, as particular cases of it, both the series' of those gentlemen and many others, some of which are fitter for this purpose than theirs are.

This method then consists in finding out such small arcs, as have for tangents some small and simple vulgar fractions (the radius being denoted by 1), and such also that some

ſome multiple of thoſe arcs ſhall differ from an arc of 45°, the tangent of which is equal to the radius, by other ſmall arcs, which alſo ſhall have tangents denoted by other ſuch ſmall and ſimple vulgar fractions. For it is evident, that if ſuch a ſmall arc can be found, ſome multiple of which has ſuch a propoſed difference from an arc of 45°, then the lengths of theſe two ſmall arcs will be eaſily computed from the general ſeries, becauſe of the ſmallneſs and ſimplicity of their tangents; after which, if the proper multiple of the firſt arc be increaſed or diminiſhed by the other arc, the reſult will be the length of an arc of 45°, or $\frac{1}{8}$th of the circumference. And the manner in which I diſcover ſuch arcs is thus:

Let T, *t*, denote any two tangents, of which T is the greater, and *t* the leſs; then it is known, that the tangent of the difference of the correſponding arcs is equal to $\frac{T-t}{1+Tt}$. Hence, if *t*, the tangent of the ſmaller arc, be ſucceſſively denoted by each of the ſimple fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. the general expreſſion for the tangent of the difference between the arcs will become reſpectively $\frac{2T-1}{2+T}$, $\frac{3T-1}{3+T}$, $\frac{4T-1}{4+T}$, $\frac{5T-1}{5+T}$, &c.; ſo that if T be expounded by any given number, then theſe expreſſions will give the tangent of the difference of the arcs in known numbers, according to the values of *t*, ſeverally aſſumed reſpectively. And if, in the firſt place, T be equal to 1, the tangent of 45°, the foregoing expreſſions will give the tangent of an arc, which is equal to the difference between that of 45° and the firſt arc; or that, of which the tangent

tangent is one of the numbers $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. Then if the tangent of this difference, juſt now found, be taken for T, the ſame expreſſions will give the tangent of an arc, which is equal to the difference between the arc of 45° and the double of the firſt arc. Again, if for T we take the tangent of this laſt found difference, then the foregoing expreſſions will give the tangent of an arc, equal to the difference between that of 45° and the triple of the firſt arc. And again taking this laſt found tangent for T, the ſame theorem will produce the tangent of an arc equal to the difference between that of 45° and the quadruple of the firſt arc; and ſo on, always taking for T the tangent laſt found, the ſame expreſſions will give the tangent of the difference between the arc of 45° and the next greater multiple of the firſt arc; or that, of which the tangent was at firſt aſſumed equal to one of the ſmall numbers $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. This operation, being continued till ſome of the expreſſions give ſuch a fit, ſmall, and ſimple fraction as is required, is then at an end, for we have then found two ſuch ſmall tangents as were required, *viz.* the tangent laſt found, and the tangent firſt aſſumed.

Here follow the ſeveral operations adapted to the ſeveral values of *t*. The letters *a*, *b*, *c*, *d*, &c. denote the ſeveral ſucceſſive tangents.

1. Take $t = \frac{1}{2}$, then the theorem $\frac{2T-1}{2+T}$ gives

$$a = \frac{1}{3}$$

$$b = \frac{-1}{7}$$

Therefore the arc of 45°, or $\frac{1}{8}$th of the circumference, is either

either equal to the ſum of the two arcs of which $\frac{1}{2}$ and $\frac{1}{3}$ are the tangents, or to the difference between the arc of which the tangent is $\frac{1}{7}$, and the double of the arc of which the tangent is $\frac{1}{2}$; that is, putting A = the arc of 45°.

$$A = \begin{cases} +\frac{1}{2} \times : 1 - \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 4^2} - \frac{1}{7 \cdot 4^3} + \frac{1}{9 \cdot 4^4} - \&c. \\ +\frac{1}{3} \times : 1 - \frac{1}{3 \cdot 9} + \frac{1}{5 \cdot 9^2} - \frac{1}{7 \cdot 9^3} + \frac{1}{9 \cdot 9^4} - \&c. \end{cases}$$

$$\text{Or, } A = \begin{cases} +1 - \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 4^2} - \frac{1}{7 \cdot 4^3} + \frac{1}{9 \cdot 4^4} - \&c. \\ -\frac{1}{7} \times : 1 - \frac{1}{3 \cdot 49} + \frac{1}{5 \cdot 49^2} - \frac{1}{7 \cdot 49^3} + \frac{1}{9 \cdot 49^4} - \&c. \end{cases}$$

And the former of theſe values of A is the ſame with that before mentioned as given by M. EULER; but the latter is much better, as the powers of $\frac{1}{49}$ converge much faſter than thoſe of $\frac{1}{9}$.

COROL. From double the former of theſe values of A ſubtracting the latter, the remainder is,

$$A = \begin{cases} +\frac{2}{3} \times : 1 - \frac{1}{3 \cdot 9} + \frac{1}{5 \cdot 9^2} - \frac{1}{7 \cdot 9^3} + \frac{1}{9 \cdot 9^4} - \&c. \\ +\frac{1}{7} \times : 1 - \frac{1}{3 \cdot 49} + \frac{1}{5 \cdot 49^2} - \frac{1}{7 \cdot 49^3} + \frac{1}{9 \cdot 49^4} - \&c. \end{cases}$$

which is a much better theorem than either of the former.

2. If t be taken $= \frac{1}{3}$, then the expreſſion $\frac{3T - 1}{3 + T}$ gives,

$$a = \frac{1}{2}$$

$$b = \frac{1}{7}$$

Here the value of $a = \frac{1}{2}$ gives the ſame expreſſion for the value of A as the firſt in the foregoing caſe, and the value

 of

of $b=\frac{1}{7}$ gives the value of A the very fame as in the corollary to the cafe above.

3. Taking $t=\frac{1}{4}$, the expreffion $\frac{4T-1}{4+T}$ gives

$$a = \frac{3}{5}$$
$$b = \frac{7}{23}$$
$$c = \frac{5}{99}$$
$$d = -\frac{79}{401}$$

Here it is evident, that the value of $c=\frac{5}{99}$ is the fitteft number afforded by this cafe; and from it it appears, that the arc of 45° is equal to the fum of the arc of which the tangent is $\frac{5}{99}$, and the triple of the arc of which the tangent is $\frac{1}{4}$.

$$\text{Or that A} = \begin{cases} +\frac{3}{4} \times : 1 - \frac{1}{3.16} + \frac{1}{5.16^2} - \frac{1}{7.16^3} + \frac{1}{9.16^4} - \&c. \\ +\frac{5}{99} \times : 1 - \frac{5^2}{3.99^2} + \frac{5^4}{5.99^4} - \frac{5^6}{7.99^6} + \frac{5^8}{9.99^8} - \&c. \end{cases}$$

Which is the beft theorem that we have yet found, becaufe that the number 99 refolves into the two eafy factors 9 and 11.

4. Let now t be taken $=\frac{1}{5}$, and the expreffion $\frac{5T-1}{5+T}$ will give

$$a = \frac{2}{3}$$
$$b = \frac{7}{17}$$
$$c = \frac{9}{46}$$
$$d = \frac{-1}{239}$$

Where

Where it is evident, that the laft number, or the value of *d*, is the fitteft of thofe produced in this cafe, and from which it appears, that the arc of 45° is equal to the difference between the arc of which the tangent is $\frac{1}{239}$, and quadruple the arc of which the tangent is $\frac{1}{5}$. Or that

$$A = \begin{cases} +\frac{4}{5} \times : 1 - \frac{1}{3 \cdot 5^2} + \frac{1}{5 \cdot 5^4} - \frac{1}{7 \cdot 5^6} + \frac{1}{9 \cdot 5^8} - \&c. \\ -\frac{1}{239} \times : 1 - \frac{1}{3 \cdot 239^2} + \frac{1}{5 \cdot 239^4} - \frac{1}{7 \cdot 239^6} + \frac{1}{9 \cdot 239^8} - \&c. \end{cases}$$

Which is the very theorem that was invented by Mr. MACHIN, as we have before mentioned.

5. Take now $t = \frac{1}{6}$, and the expreffion $\frac{6T - 1}{6 + T}$ will give

$$a = \frac{5}{7}$$

$$b = \frac{23}{47}$$

$$c = \frac{91}{305}$$

$$d = \frac{241}{1921}$$

$$e = \frac{-475}{11767}$$

Of which numbers none, it is evident, are fit for our purpofe.

 6. Again,

6. Again, take $t=\frac{1}{7}$, and the expreffion $\frac{7T-1}{7+T}$ will give

$$a = \frac{3}{4}$$
$$b = \frac{17}{31}$$
$$c = \frac{11}{28}$$
$$d = \frac{49}{205}$$
$$e = \frac{69}{742}$$
$$f = \frac{-259}{5263}$$

Neither are any of thefe fit numbers for our purpofe.

7. In like manner take $t=\frac{1}{8}$, fo fhall $\frac{8T-1}{8+T}$ give

$$a = \frac{7}{9}$$
$$b = \frac{47}{79}$$
$$c = \frac{297}{679}$$
$$d = \frac{1697}{5729}$$
$$e = \frac{7847}{47529}$$
$$f = \frac{14047}{388079}$$

8. And

8. And if t be taken $=\frac{1}{9}$, the expreſſion $\frac{9T-1}{9+T}$ will give

$$a = \frac{4}{5}$$
$$b = \frac{31}{49}$$
$$c = \frac{115}{236}$$
$$d = \frac{799}{2239}$$
$$e = \frac{2467}{10475}$$
$$f = \frac{11809}{96751}$$
$$g = \frac{4765}{441284}$$

9. Alſo, if we take $t=\frac{1}{10}$, the expreſſion $\frac{10T-1}{10+T}$ will give

$$a = \frac{9}{11}$$
$$b = \frac{79}{119}$$
$$c = \frac{671}{1269}$$
$$d = \frac{5441}{13361}$$
$$e = \frac{41049}{139051}$$
$$f = \frac{271439}{1431559}$$
$$g = \frac{1282831}{14587029}$$

10. Farther

10. Farther, if we take $t = \frac{1}{11}$, the expression $\frac{11T-1}{11+T}$ will give

$$a = \frac{5}{6}$$

$$b = \frac{49}{71}$$

$$c = \frac{234}{415}$$

$$d = \frac{2159}{4799}$$

$$e = \frac{9475}{27474}$$

$$f = \frac{76751}{311689}$$

$$g = \frac{266286}{1752665}$$

$$h = \frac{1176481}{19545601}$$

11. Lastly, if we take $t = \frac{1}{12}$, the expression $\frac{12T-1}{12+T}$ gives

$$a = \frac{11}{13}$$

$$b = \frac{113}{167}$$

$$c = \frac{41}{73}$$

$$d = \frac{419}{917}$$

$$e = \frac{4111}{11423}$$

$$f = \frac{37909}{141187}$$

$$g = \frac{313721}{1732153}$$

$$h = \frac{2032499}{21099557}$$

$$i = \frac{3290431}{255227183}$$

Here

Here it is evident, that none of these latter cases afford any numbers that are fit for this purpose. And to try any other fractions less than $\frac{1}{12}$ for the value of t, does not seem likely to answer any good purpose, especially, as the divisors, after 12, become too large to be managed in the easy way of short division in one line.

By the foregoing means it appears, that we have discovered five different forms of the value of A, or $\frac{1}{4}$th of the semi-circumference, all of which are very proper for readily computing its length; *viz.* three forms in the first case and its corollary, one in the third case, and one in the fourth case. Of these, the first and last are the same as those invented by EULER and MACHIN respectively, and the other three are quite new, as far as I know.

But another remarkable excellency attending the first three of the before mentioned series is, that they are capable of being changed into others which not only converge still faster, but in which the converging quantity shall be $\frac{1}{10}$, or some multiple or sub-multiple of it; and so the powers of it raised with the utmost ease. The series', or theorems, here meant are these three:

$$\text{1st, A} = \begin{cases} +\frac{1}{2} \times : 1 - \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 4^2} - \frac{1}{7 \cdot 4^3} + \frac{1}{9 \cdot 4^4}, \text{ \&c.} \\ +\frac{1}{3} \times : 1 - \frac{1}{3 \cdot 9} + \frac{1}{5 \cdot 9^2} - \frac{1}{7 \cdot 9^3} + \frac{1}{9 \cdot 9^4}, \text{ \&c.} \end{cases}$$

$$\text{2dly, A} = \begin{cases} +1 - \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 4^2} - \frac{1}{7 \cdot 4^3} + \frac{1}{9 \cdot 4^4}, \text{ \&c.} \\ -\frac{1}{7} \times : 1 - \frac{1}{3 \cdot 49} + \frac{1}{5 \cdot 49^2} - \frac{1}{7 \cdot 49^3} + \frac{1}{9 \cdot 49^4}, \text{ \&c.} \end{cases}$$

$$3\text{dly, A}=\begin{cases}+\frac{2}{3}\times:1-\frac{1}{3\cdot9}+\frac{1}{5\cdot9^2}-\frac{1}{7\cdot9^3}+\frac{1}{9\cdot9^4},\ \&c.\\+\frac{1}{7}\times:1-\frac{1}{3\cdot49}+\frac{1}{5\cdot49^2}-\frac{1}{7\cdot49^3}+\frac{1}{9\cdot49^4},\ \&c.\end{cases}$$

Now if each of these be transformed, by means of the differential series in cor. 3. p. 64. of the late Mr. THOMAS SIMPSON's Mathematical Dissertations, they will become of these very commodious forms, *viz.*

$$1\text{st, A}=\begin{cases}+\frac{4}{10}\times:1+\frac{4}{3\cdot10}+\frac{8\alpha}{5\cdot10}+\frac{12\beta}{7\cdot10}+\frac{16\gamma}{9\cdot10},\ \&c.\\+\frac{3}{10}\times:1+\frac{2}{3\cdot10}+\frac{4\alpha}{5\cdot10}+\frac{6\beta}{7\cdot10}+\frac{8\gamma}{9\cdot10},\ \&c.\end{cases}$$

$$2\text{dly, A}=\begin{cases}+\frac{4}{5}\times:1+\frac{4}{3\cdot10}+\frac{8\alpha}{5\cdot10}+\frac{12\beta}{7\cdot10}+\frac{16\gamma}{9\cdot10},\ \&c.\\-\frac{7}{50}\times:1+\frac{4}{3\cdot100}+\frac{8\alpha}{5\cdot100}+\frac{12\beta}{7\cdot100}+\frac{16\gamma}{9\cdot100},\ \&c.\end{cases}$$

$$3\text{dly, A}=\begin{cases}+\frac{6}{10}\times:1+\frac{2}{3\cdot10}+\frac{4\alpha}{5\cdot10}+\frac{6\beta}{7\cdot10}+\frac{8\gamma}{9\cdot10},\ \&c.\\+\frac{7}{50}\times:1+\frac{2}{3\cdot50}+\frac{4\alpha}{5\cdot50}+\frac{6\beta}{7\cdot50}+\frac{8\gamma}{9\cdot50},\ \&c.\end{cases}$$

Where α, β, γ, &c. denote always the preceding terms in each series.

Now it is evident, that all these latter series, are much easier than the former ones, to which they respectively correspond; for, because of the powers of 10 here concerned, we have little more to do than to divide by the series of odd numbers 1, 3, 5, 7, 9, &c.

Of all these three forms the second is the fittest for computing the required proportion; because that, of the two series' of which it consists, the several terms of the one are found from the like terms of the other, by dividing theſe latter by 10 and its ſeveral ſucceſſive powers 100, 1000, &c.; that is, the terms of the one conſiſt of the ſame figures as the terms of the other, only removed a certain number of places farther towards the right, in the decuple ſcale of numbers; and the number of places, by which they muſt be removed, is the ſame as the diſtance of each term from the firſt term of the ſeries, *viz.* in the ſecond term the figures muſt be moved one place lower, in the third term two, in the fourth term three, &c. ſo that the latter ſeries will conſiſt of but about half the number of the terms of the former. Thus, then, this method may be ſaid to effect the buſineſs by one ſeries only, in which there is little more to do, than to divide by the ſeveral numbers 1, 3, 5, 7, &c.; for as to the multiplications by the numbers in the numerators of the terms, after they become large, they are eaſily performed by barely multiplying by the number two, and ſubtracting one number from another: for ſince every numerator is leſs by two than the double of its denominator, if d denote any denominator (excluſive always of the powers of 10) then the co-efficient of that term is $\frac{2d-2}{d}$, or $2-\frac{2}{d}$, by which the preceding term is to be multiplied; to do which, therefore, multiply it by two, that is double it, and divide that double by the diviſor d, and ſubtract the quotient from the ſaid double.

By a pretty exact eſtimate, which I have made, of the proportion of the trouble or time in computing the circumference by this middle form of the value of A, and by Mr. MACHIN's theorem, I have found, that the computation by his method requires about $\frac{1}{8}$th or $\frac{1}{10}$th more time than by mine. And its advantage over any of the ſeries' invented by EULER or others, is ſtill much more conſiderable.